



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

two solutions, if real, may be found by means of a circle on $\alpha\omega$ as diameter, as indicated in the figure.¹

II. SUPPLEMENTARY NOTE ON THE IRRATIONALITY OF CERTAIN TRIGONOMETRIC FUNCTIONS.

By R. S. UNDERWOOD, Alabama Polytechnic Institute.

THEOREM: *When an angle is rationally expressible in degrees and not a multiple of 30° or 45° , its trigonometric functions are irrational.*

The tangent of any rationally expressible angle except one of the form treated below may be proved irrational by a method so closely analogous to that used in my former paper that the details are omitted here. There remain angles of the form $(m/n)^\circ$, where m/n is an irreducible fraction and where $m = k45$.

In the expansion

$$\tan n\theta = \frac{n(n-1)(n-2)}{3!} \tan^3 \theta + \dots \pm \tan^n \theta \text{ (or } \pm n \tan^{n-1} \theta, \text{ when } n \text{ is even)}$$

$$1 - \frac{n(n-1)}{2!} \tan^2 \theta + \dots \pm n \tan^{n-1} \theta \text{ (or } \tan^n \theta, \text{ when } n \text{ is even)}$$

the substitution $\tan n\theta = \pm 1$ gives an n th degree equation whose only possible rational roots (± 1) are excluded by the conditions of the problem. When $\tan n\theta = 0$ or $\pm \infty$, it is evident that the roots obtained by equating the numerator and denominator respectively to zero are either integers or the reciprocals of integers. Furthermore, if $\tan \theta$ is rational, $\tan 2\theta$, when it is finite, is rational. But if $\tan \theta = k$ or $1/k$, k being an integer, $\tan 2\theta = \pm 2k/(1-k^2)$, which is neither an integer nor the reciprocal of an integer, since the denominator lacks the factor k . Hence $\tan \theta$ is irrational.

The above theorem is interesting in that it brings out in a striking way for pedagogical purposes the great preponderance of irrational over rational numbers. Divide a right angle into n equal parts. Then there are only three distinct angles in all this infinite family which possess rational trigonometric functions.

Incidentally I have shown that the algebraic equations of the form (n odd)

$$x^{n-1} - \frac{n(n-1)}{2!} x^{n-3} + \dots \pm n = 0,$$

and

$$nx^{n-1} - \frac{n(n-1)(n-2)}{3!} x^{n-3} + \dots \pm 1 = 0,$$

have no rational roots.

¹ General Bixby would be glad to hear from any engineer, physicist or mathematician who is frequently required to solve higher equations, what method he uses, how much time is required, and what accuracy is deemed necessary.